CIVIL ENGINEERING-CE

GATE / PSUs

STUDY MATERIAL

THEORY OF STRUCTURE

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CONTENT

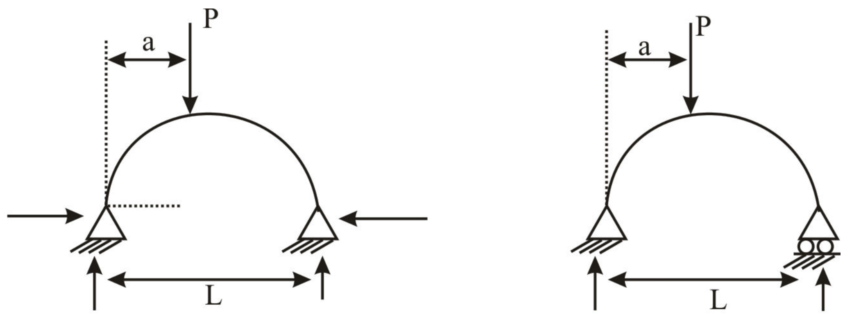
CHAPTER-1: ARCHES

Introduction:

→ Inverted cable is Arch and they are determinant.
→ As cables are in pure tension, arches will be in a state of pure compression under the action of external loads.
→ These compressive forces @ end hinges produce horizontal and vertical reactions.
→ The difference between the beam and the arch is that in the case of the arch, the horizontal thrust induced at support provides a hogging moment at any section in the arch.
→ If at a section, the hogging moment produced by a horizontal thrust "H". Then, actual moment will be reduced as.

$$\left.\begin{array}{c}\text{Actual moment at}\\ \text{section } x-x\end{array}\right\} = \begin{array}{l}\text{Bending moment due}\\ \text{to external loads at}\\ \text{that section}\end{array} - \begin{array}{l}H-\text{moment at}\\ \text{the section } x-x\end{array}$$

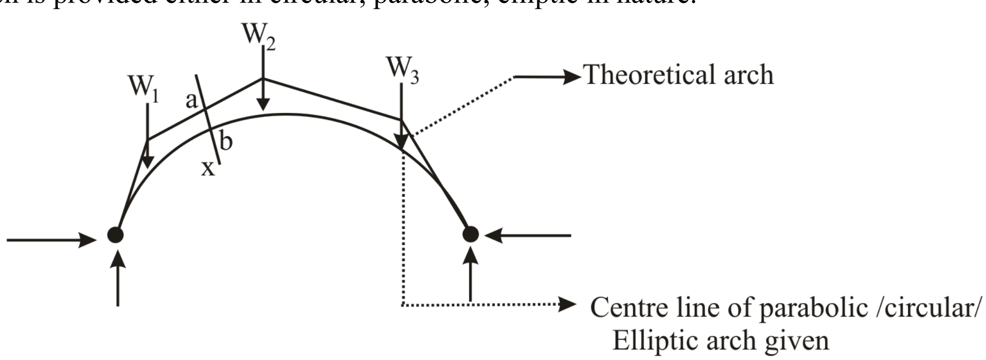
→ Under the similar load conditions thus sectional requirement for an arch is less than that of a beam.
→ It is important to appreciate the point that the definition of an arch is a structural one, not geometrical.
→ A structure is classified as an arch because of the way it supports the lateral load and the development of horizontal reaction.



(a) Arch (b) Bean

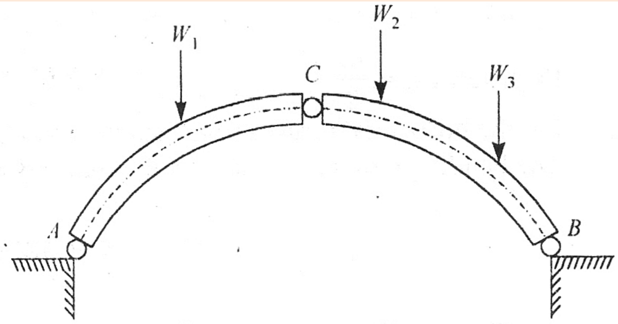
In the above example, figure (b) is not an arch, as there is no horizontal reaction is developed.

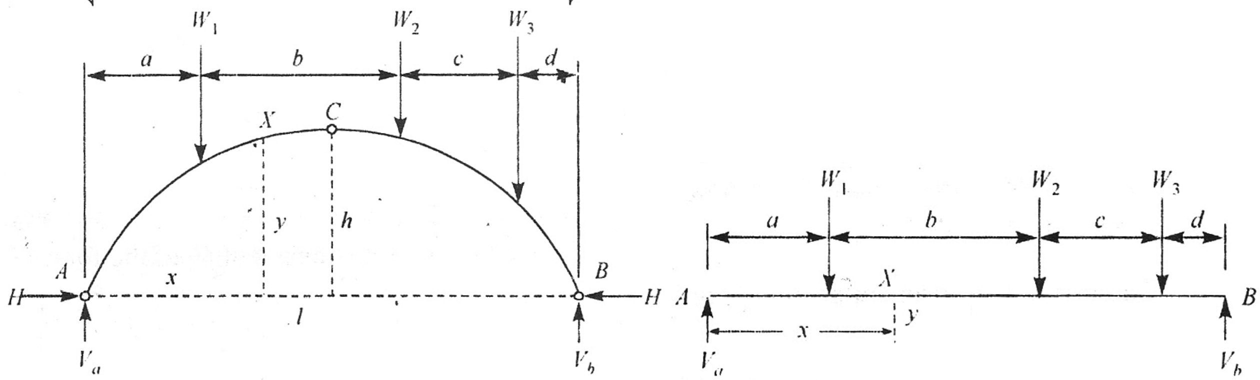
→ Theoretical arch: - If the arch is drawn such that it represents the thrust in various parts of arch, then it is called as theoretical arch. It is also called as LINE OF THRUST.
→ It is not possible to give an arch it's theoretical shape because it changes for varying load positions.
→ Therefore, arch is provided either in circular, parabolic, elliptic in nature.



→ In the above figure, thus there exists an intercept between theoretical arch and actual arch. This intercept acts as eccentricity and thus produces a moment.
→ Therefore at any section in the arch. There exists an horizontal force, vertical force ζ Bending moment.
→ There are mainly three types of arches that are commonly used in practice: three hinged arch, two hinged arch ζ fixed arch.

Three Hinged Arches:

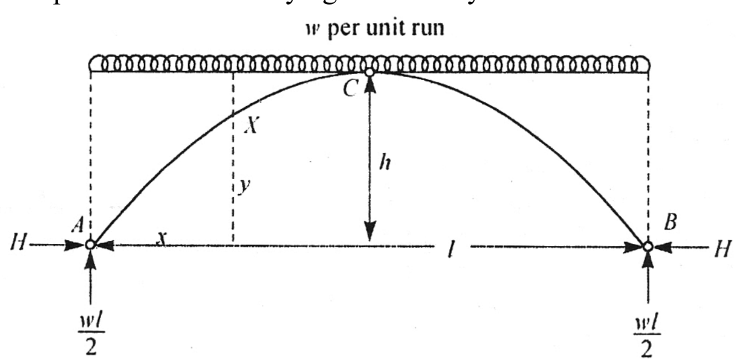




- Three Hinged arches are statically determinate arches.
- As shown above, the arch is hinged at ‘A’, ‘B’ and ‘C’.
- The horizontal distance between the lower hinges ‘A’ and ‘B’ is called **‘span’ of** the arch.
- The hinges ‘A’ and ‘B’ may or may not be at the same level.
- When two lower hinges (A and B) are at the same level the height of the crown (highest point of the arch) above the level of the lower hinges is called **rise** of the arch.
- When the hinges are different level, Rise is taken as the distance between bottom hinge to top portion of the arch.

SOME ILLUSTRATIONS:

Case I: A three hinged arch of span l and rise h carrying a uniformly distributed load of w per unit run over the span.

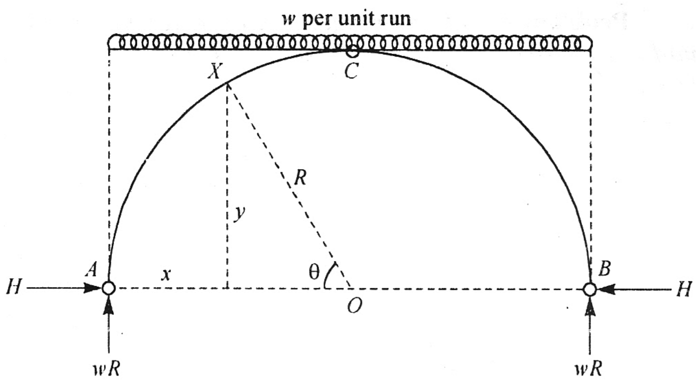


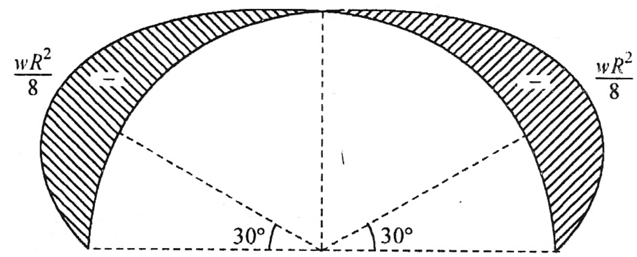
- Horizontal thrust at each support, $H = \dfrac{wl^2}{8h}$
- Bending moment at any section of the arch is zero, $\boxed{M = 0}$
- The equation of the arch with end ‘A’ as origin is given by: $\boxed{y = \dfrac{4h}{l^2}x(l - x)}$

- The angle of slope (θ) at any given section will be given by: $\tan\theta = \frac{dy}{dx} = \frac{4y_c}{l^2}(l-2x)$

$$\Rightarrow \boxed{\theta = \tan^{-1}\left[\frac{4y_c}{l^2}(l-2x)\right]}$$

Case II: A three hinged semicircular arch of the radius 'R' carrying a uniformly distributed load of *w* per unit run over the whole span.



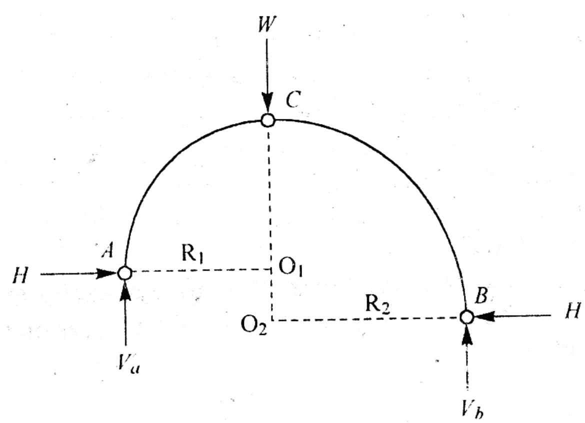


- Horizontal thrust at each end, $H = \frac{wR}{2}$.
- Bending moment at any section X-X making an angle 'θ' with the horizontal is given by:

$\boxed{M_x = -\frac{wR^2}{2}[\sin\theta - \sin^2\theta]}$ (hogging moment)

- Maximum bending moment, $\boxed{M_{max.} = \frac{wR^2}{8}}$ At an angle $\boxed{\theta = 30°}$ i.e. Distance of the point of maximum bending moment from the crown $= R\cos 30° = \frac{R\sqrt{3}}{2}$.

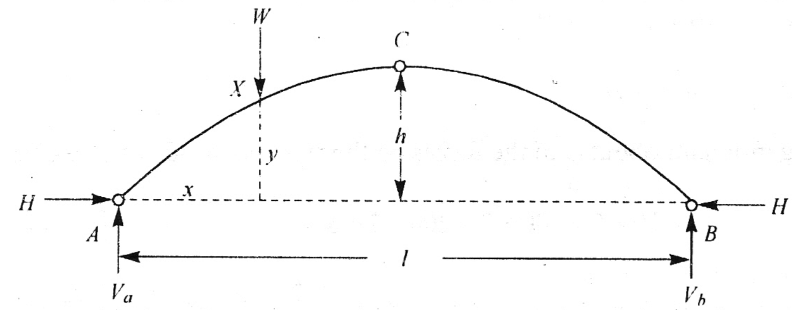
Case III: A three hinged arch consisting of two quadrant parts AC and CB of radii R_1 and R_2 carrying a concentrated load 'W' on the crown.



- Horizontal thrust at each end, $\boxed{H = \frac{W}{2}}$
- Reactions at each end equal to the horizontal thrust

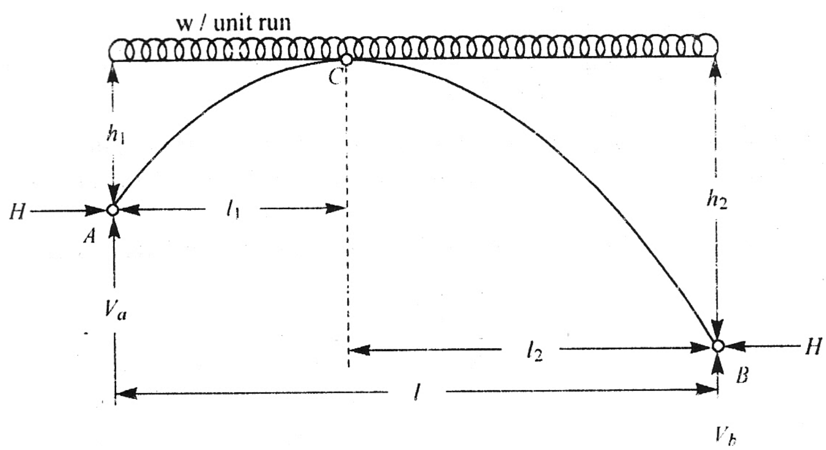
$$\boxed{V_a = V_b = H = \frac{W}{2}}$$

Case IV:A symmetrical three hinged parabolic arch of span 'l' and rise 'h' carrying a point load 'W' which may be placed anywhere on the span.



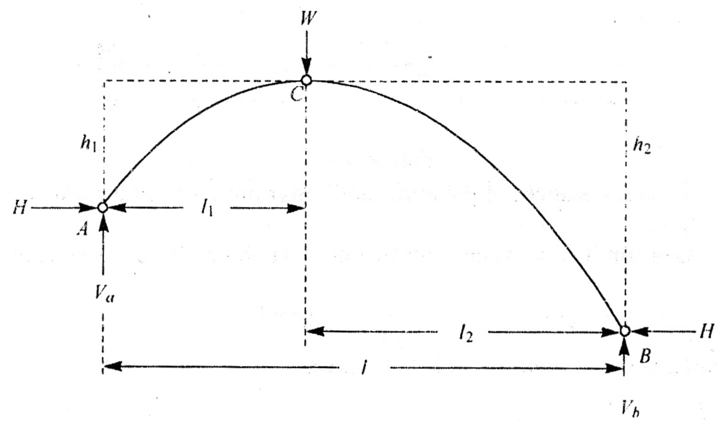
- Horizontal thrust at each end, $\boxed{H = \frac{Wx}{2h}}$
- Maximum Bending momentoccurs under the load.
- For the condition of absolute maximum bending moment, $\boxed{x = \frac{l}{2\sqrt{3}}}$ on either side of the crown

Case V: A three hinged parabolic arch of span 'l' having its abutments at depth h_1 and h_2 below the crowns carrying a uniformly distributed load of w per unit run over the whole span.



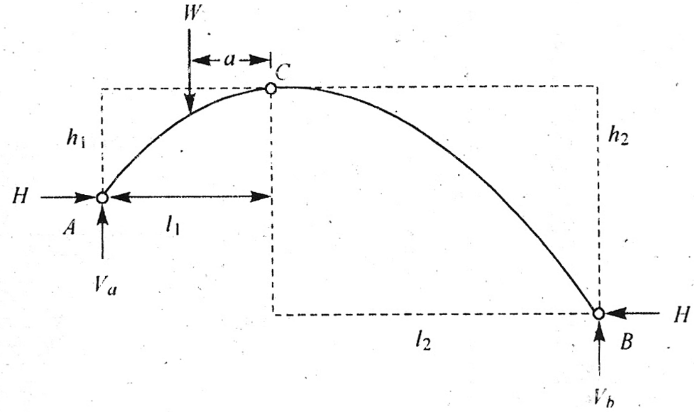
- Horizontal thrust at each end, $H = \dfrac{wl^2}{2\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}$

Case VI: A three hinged parabolic arch of span 'l' having its abutments 'A' and 'B' at depths h_1 and h_2 below the crown carrying a concentrated load 'W' at the crown.



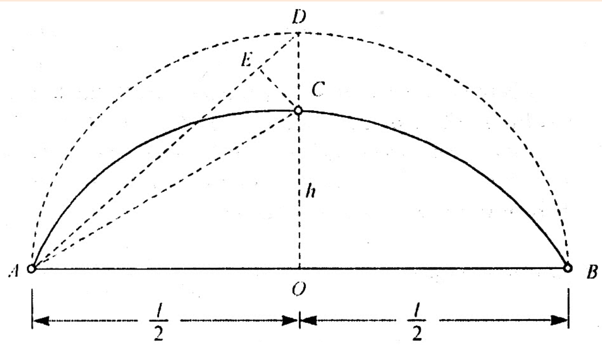
- Horizontal thrust at each end, $H = \dfrac{Wl}{\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}$

Case VII: A three hinged parabolic arch of span 'l' having its abutments 'A' and 'B' at depths h_1 and h_2 below the crown carrying a concentrated load 'W' at a distance 'a' from crown.



- Horizontal thrust at each end. $H = \dfrac{Wl_2(l_1 - a)}{h_1 l_2 + h_2 l_1}$

Temperature effect on three-hinged arch:



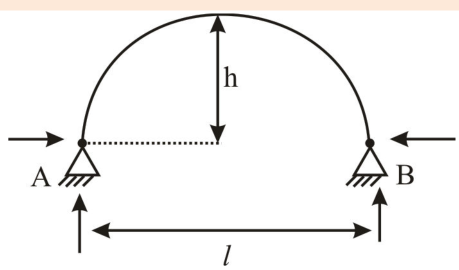
- Rise in temperature increases the length of the arch. Since the ends A and B do not move and since the hinge *C* is not connected to any permanent object, the crown hinge will rise from C to D. 'AD' represents the new position of AC.
- Increase in the rise of arch $= CD = \delta = \dfrac{l^2 + 4h^2}{4h} \propto T$,

  Where, T = change in temp. (ºC)

  $\propto$ = Co-efficient of linear expansion
- Due to temperature change, stresses are not produced in the arch, but the horizontal thrust changes.

  $\boxed{\dfrac{dH}{H} = -\dfrac{dh}{h}}$ i.e. Horizontal thrust decreases due to rise in temperature.

Two hinged arches:



- Two hinged arch is statically indeterminate to 1 degree.

- No. of unknowns = 4 and equations of equilibrium = 3. Therefore, static indeterminacy = 1
- Vertical reactions can be computed by taking moments about hinges equal to zero.
- Horizontal reactions i.e., Horizontal Thrust (H) can be found by using one of the following methods.

(a.) By assuming horizontal displacement of hinge B due to bending moments on all the elements from A to B of the curved bar equal to zero.
(b.) By using the principle of minimum strain energy.

- Both methods given same result, but using principle of minimum strain energy is a quick approach.
- Strain energy stored in the whole arch, $\boxed{U_i = \int (M - Hy)^2 \frac{ds}{2EI}}$ where, M = B.M. of beam at section X-X
- By First theorem of Castigliano, the horizontal thrust can be obtained using $\boxed{\frac{\partial U_i}{\partial H} = 0}$

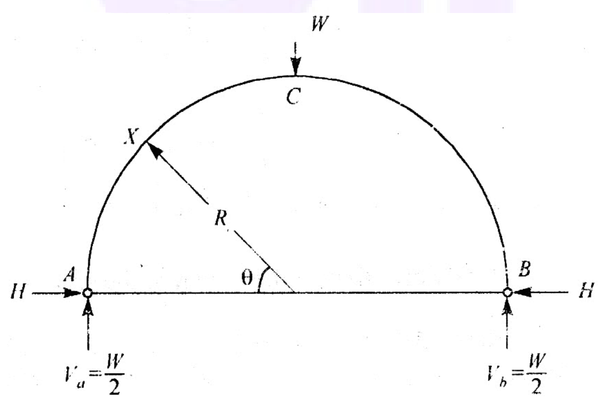
Horizontal thrust, $H = \dfrac{\int \frac{Myds}{EI}}{\int \frac{y^2 ds}{EI}}$

If flexural rigidity of arch is uniform, $\boxed{H = \dfrac{\int Myds}{\int y^2 ds}}$

- For a parabolic arches, it can also be given as: $\boxed{H = \dfrac{\int Mydx}{\int y^2 dx}}$

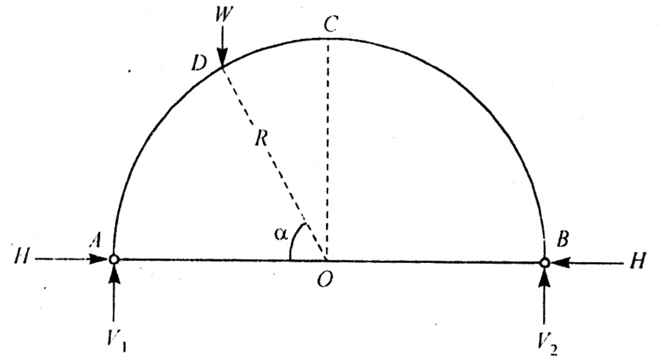
SOME ILLUSTRATIONS:

Case I: A two-hinged semicircular arch of radius 'R' carrying a concentrated load 'W' at the crown. Flexural rigidity (EI) is constant.



- Horizontal thrust, $\boxed{H = \frac{W}{\pi}}$
- Horizontal thrust is independent of Radius of the arch.
- Vertical deflection of the crown, $\delta = \frac{WR^3}{8\pi EI}(3\pi^2 - 8\pi - 4)$

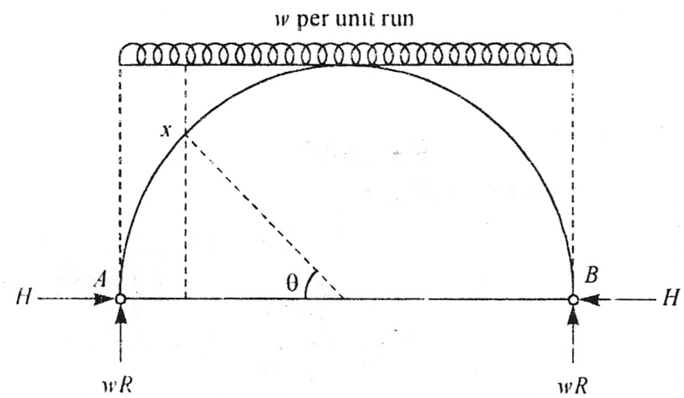
Case II: A two-hinged semicircular arc of radius 'R' carrying a load W at a section the radius vector corresponding to which makes an angle $\propto$ with the horizontal. EI is constant.



- Horizontal thrust, $\boxed{H = \frac{W}{\pi}\sin^2 \alpha}$
- If there are loads W_1, W_2, W_3..... at an angle $\propto_1, \propto_2, \propto_3$ .... (Less than or equal to 90º); Horizontal thrust, H

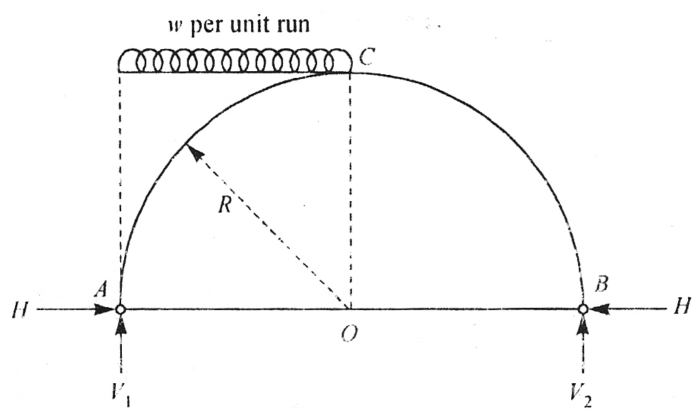
$$\boxed{H = \sum \frac{W_i}{\pi}\sin^2 \propto_i}$$

Case III: A two hinged semicircular arch of radius R carrying a uniformly distributed load w per unit run over the whole span. EI = constant.



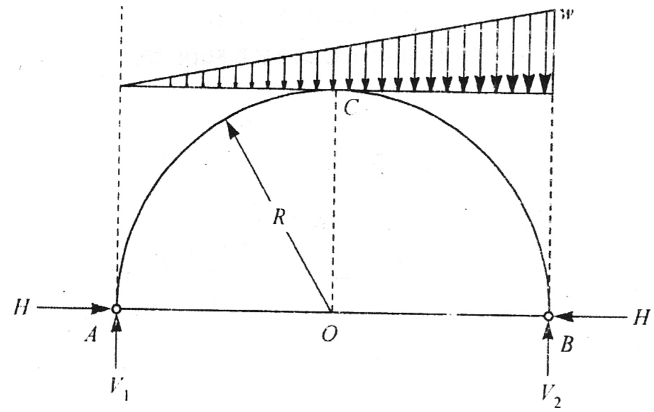
- Horizontal thrust, $H = \frac{4}{3}.\frac{wR}{\pi}$

Case IV: A two hinged semicircular arch carrying a uniformly distributed load of w per unit run over the left half of its span. EI = constant.



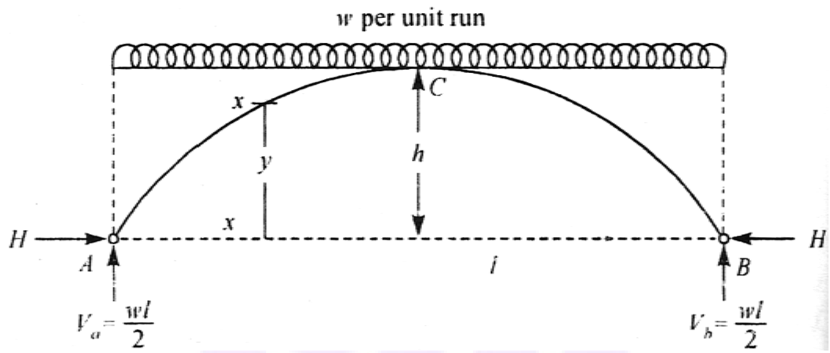
- Horizontal thrust, $H = \frac{2}{3}.\frac{WR}{\pi}.$

Case V: A two-hinged semicircular arch of radius R carrying a distributed load uniformly varying from zero at the left end to w per unit run at the right end. EI = constant



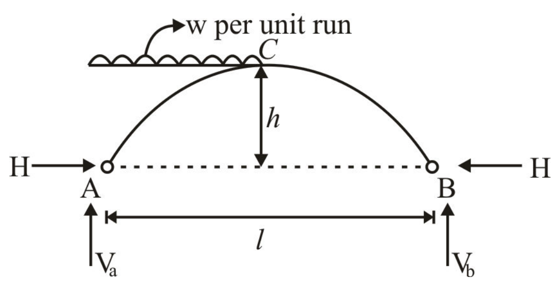
➢ Horizontal thrust, $H = \frac{4}{3}.\frac{wR}{\pi}.$

Case VI: A two-hinged parabolic arch of span *l* and rise *h* carrying a uniformly distributed load *w* per unit run over the whole span .*EI* = constant



➢ Horizontal thrust, $H = \frac{wl^2}{8h},$

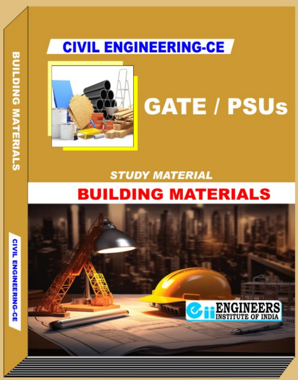
Case VII: A two-hinged parabolic arch carrying a u.d.l. of *w* per unit run on its left half of the span.
EI = constant

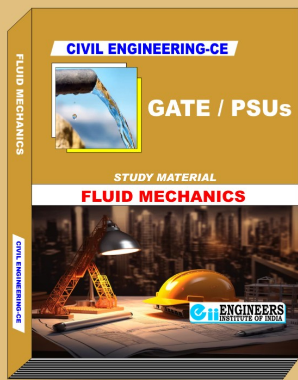


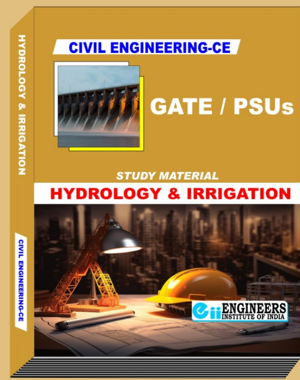
➢ Horizontal thrust, $H = \frac{wl^2}{16h}.$

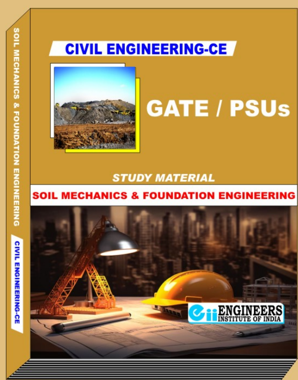
Case VIII: A two hinged parabolic arch of span '*l*' and rise '*h*' carrying a load varying uniformly from zero at the left end to *w* per unit run at right end. *EI* = constant**.**

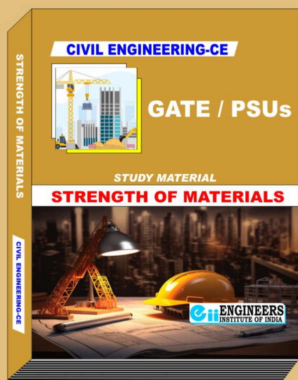
Published Books

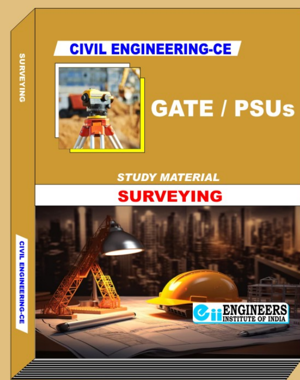


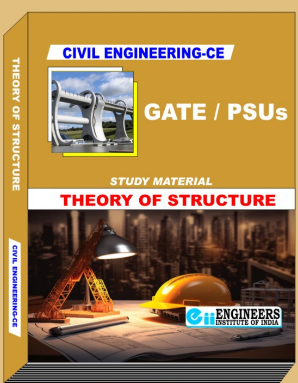


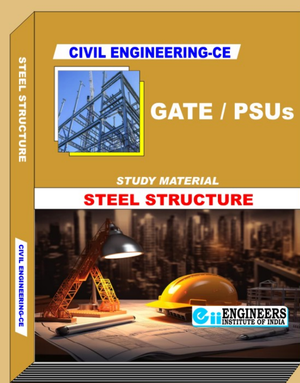












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